Polar Coordinates and Graphs

3) Leaves rose

$r = a\cos \mathbf{n}\theta$, $r = a\sin \mathbf{n}\theta$

$r = a\sin \mathbf{3}\theta$, $r = a\cos \mathbf{3}\theta\}$ 3 Leafed rose

$r = a\sin \mathbf{2}\theta$, $r = a\cos \mathbf{2}\theta\}$ 4 Leafed rose

$r^2 = a^2 \cos \mathbf{2}\theta$

1- Number of leaves:

$r = a\cos \mathbf{n}\theta$, $r = a\sin \mathbf{n}\theta$

a) If n even $\rightarrow$ No. of leaves $=2n$
b) If n odd$\rightarrow$ No. of leaves $=n$

2- The major axes for the first leaf :

a) $\cos \mathbf{n}\theta = 1 \Rightarrow n\theta = 0 \Rightarrow \theta = 0$

b) $\sin \mathbf{n}\theta = 1 \Rightarrow n\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{2n}$

3- Limit for the first leaf (begin and end)

a) $\cos \mathbf{n}\theta = 0 \Rightarrow \begin{cases} n\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{2n} \\ n\theta = \frac{-\pi}{2} \Rightarrow \theta = \frac{-\pi}{2n} \end{cases}$

b) $\sin \mathbf{n}\theta = 0 \Rightarrow \begin{cases} n\theta = 0 \Rightarrow \theta = 0 \\ n\theta = \pi \Rightarrow \theta = \frac{\pi}{n} \end{cases}$

4- Complete drawing the other leaves: (360/No. of leaves) from major axes.

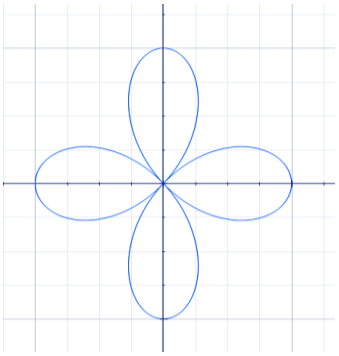
Ex.1: $r = \cos 2\theta$

1- No. of leaves = 4

2- $\cos 2\theta = 1 \Rightarrow 2\theta = 0 \Rightarrow \theta = 0$

3- $\cos 2\theta = 0 \Rightarrow \begin{cases} 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} \\ 2\theta = \frac{-\pi}{2} \Rightarrow \theta = \frac{-\pi}{4} \end{cases}$

4- $\frac{360}{4} = \frac{\pi}{2}$, this mean every $\frac{\pi}{2}$ leaf repeated.



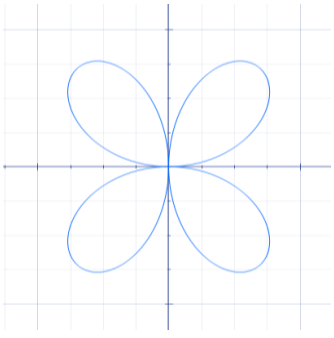
Ex.2: $r = \sin 2\theta$

1- No. of leaves = 4

2- $\sin 2\theta = 1 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$

3- $\sin 2\theta = 0 \Rightarrow \begin{cases} 2\theta = 0 \Rightarrow \theta = 0 \\ 2\theta = \pi \Rightarrow \theta = \frac{\pi}{2} \end{cases}$

4- $\frac{360}{4} = \frac{\pi}{2}$, this mean every $\frac{\pi}{2}$ leaf repeated.



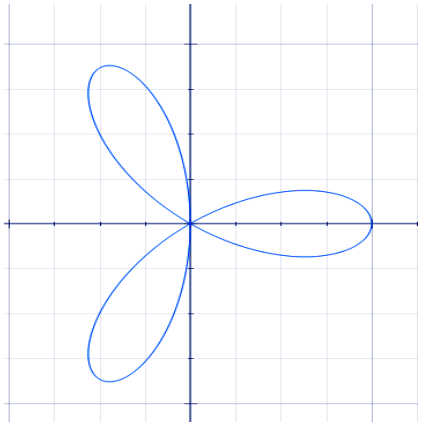
Ex.3: $r = \cos 3\theta$

1- No. of leaves = 3

2- $\cos 3\theta = 1 \Rightarrow 3\theta = 0 \Rightarrow \theta = 0$

3- $\cos 3\theta = 0 \Rightarrow \begin{cases} 3\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{6} \\ 3\theta = \frac{-\pi}{2} \Rightarrow \theta = \frac{-\pi}{6} \end{cases}$

4- $\frac{360}{3} = \frac{2\pi}{3}$, this mean every $\frac{2\pi}{3}$ leaf repeated.



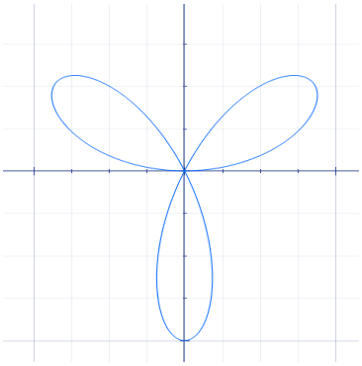
Ex.4: $r = \sin 3\theta$

1- No. of leaves = 3

2- $\sin 3\theta = 1 \Rightarrow 3\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{6}$

3- $\sin 3\theta = 0 \Rightarrow \begin{cases} 3\theta = 0 \Rightarrow \theta = 0 \\ 3\theta = \pi \Rightarrow \theta = \frac{\pi}{3} \end{cases}$

4- $\frac{360}{3} = \frac{2\pi}{3}$, this mean every $\frac{2\pi}{3}$ leaf repeated.



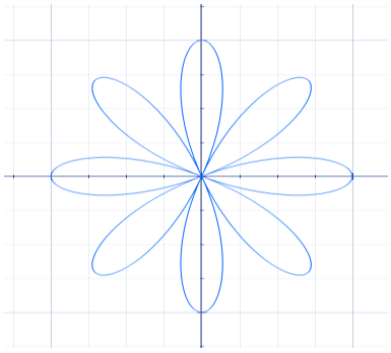
Ex.5: $r = \cos 4\theta$

1- No. of leaves = 8

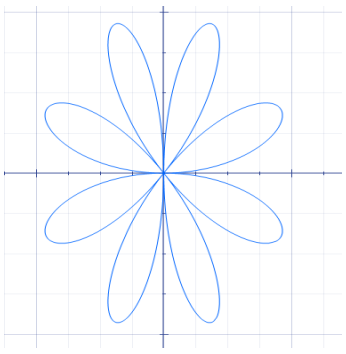
2- $\cos 4\theta = 1 \Rightarrow 4\theta = 0 \Rightarrow \theta = 0$

3- $\cos 4\theta = 0 \Rightarrow \begin{cases} 4\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{8} \\ 4\theta = \frac{-\pi}{2} \Rightarrow \theta = \frac{-\pi}{8} \end{cases}$

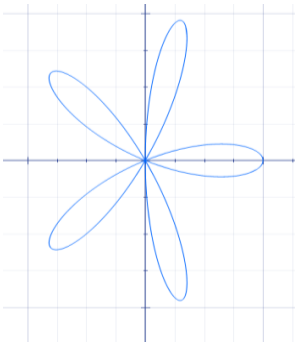
4- $\frac{360}{8} = \frac{2\pi}{8} = \frac{\pi}{4}$, this mean every $\frac{\pi}{4}$ leaf repeated.



Ex.6: $r = \sin 4\theta$



Ex.7: $r = \cos 5\theta$



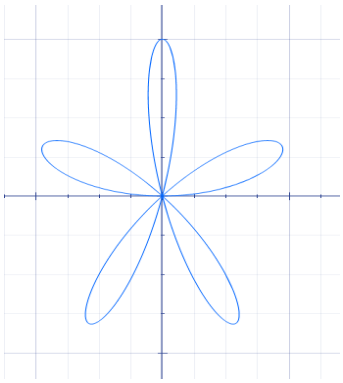
Ex.8: $r = \sin 5\theta$

1- No. of leaves = 5

2- $\sin 5\theta = 1 \Rightarrow 5\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{10}$

3- $\sin 5\theta = 0 \Rightarrow \begin{cases} 5\theta = 0 \Rightarrow \theta = 0 \\ 5\theta = \pi \Rightarrow \theta = \frac{\pi}{5} \end{cases}$

4- $\frac{360}{5} = \frac{2\pi}{5} = \frac{\pi}{4}$, this mean every $\frac{2\pi}{5}$ leaf repeated.



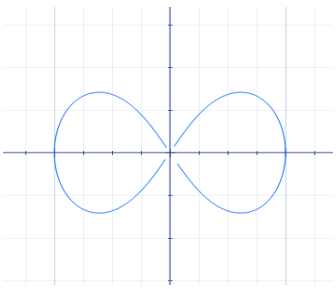
Ex.9: $r^2 = a^2 \cos 2\theta$

1- No. of leaves = 2

2- $\cos 2\theta = 1 \Rightarrow 2\theta = 0 \Rightarrow \theta = 0$

3- $\cos 2\theta = 0 \Rightarrow \begin{cases} 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} \\ 2\theta = \frac{-\pi}{2} \Rightarrow \theta = \frac{-\pi}{4} \end{cases}$

4- $\frac{360}{2} = \frac{2\pi}{2}$, this mean every π leaf repeated.



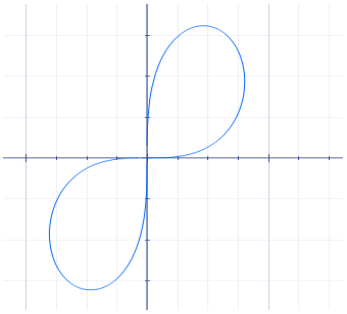
Ex.10: $r^2 = a^2 \sin 2\theta$

1- No. of leaves = 2

2- $\sin 2\theta = 1 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$

3- $\sin 2\theta = 0 \Rightarrow \begin{cases} 2\theta = 0 \Rightarrow \theta = 0 \\ 2\theta = \pi \Rightarrow \theta = \frac{\pi}{2} \end{cases}$

4- $\frac{360}{2} = \frac{2\pi}{2} = \pi$, this mean every π leaf repeated.



Problems

11) Find the area of the region enclosed by the cardioid $r^2 = 4\cos 2\theta$.

12) Find the area of the region enclosed by the cardioid $r = 12(\cos 3\theta)$.

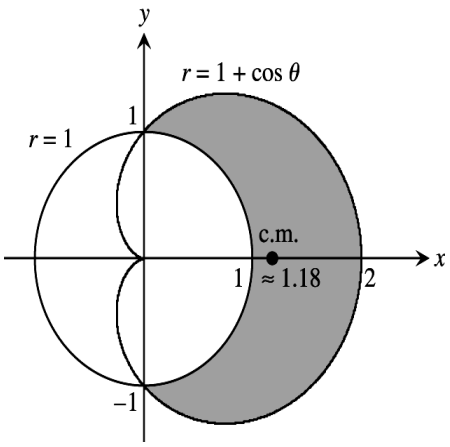
13) Find the area of the region in the first quadrant bounded by $r = 1$ **and** $r = \sin 2\theta$, $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

References:

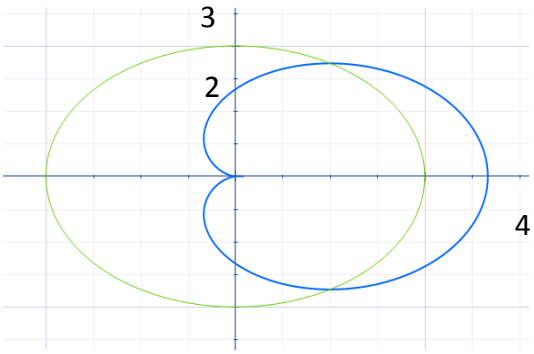
1- calculus & Analytic Geometry (Thomas).
2- Calculus (Haward Anton).
3- Advanced Mathematics for Engineering Studies **(أ. رياض احمد عزت)**

Graphs of some problems:

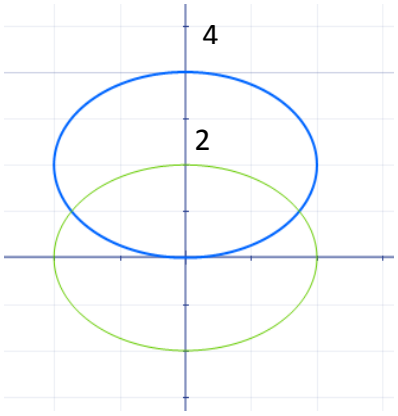
6) Find the area of the region R that lies inside the cardioid $r=(1+\cos\theta)$ and outside the circle $r=1$.



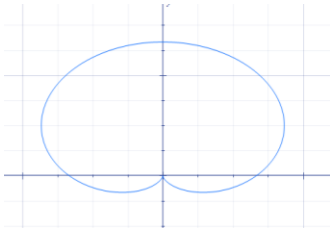
7) Find the area of the region R that lies inside the cardioid $r=2(1+\cos\theta)$ and outside the circle $r=3$.



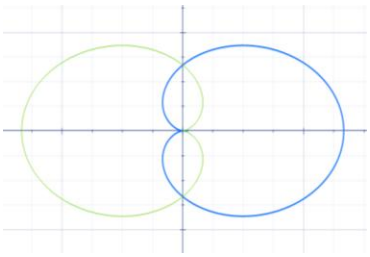
8) Find the area of the region R that lies inside the circle $r=4(\sin\theta)$ and outside the circle $r=2$.



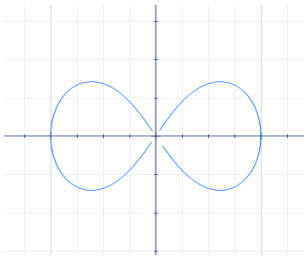
9) Find the area of the region R cut from the first quadrant by the cardioid $r = (1+\sin\theta)$.



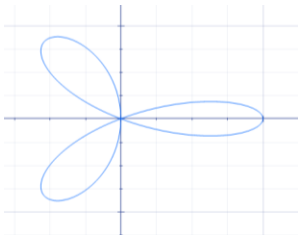
10) Find the area of the region common to the $r = (1+\cos\theta)$ and $r = (1-\cos\theta)$.



11) Find the area of the region enclosed by the cardioid $r^2 = 4\cos 2\theta$.

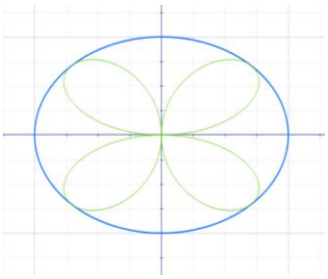


12) Find the area of the region enclosed by the $r = 12(\cos 3\theta)$.



13) Find the area of the region in the first quadrant bounded by $r = 1$ **and**

$r = \sin 2\theta$, $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$



Polar Integral: